On the Chromatic Thresholds of Hypergraphs

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Abstract

Let $\mathcal{F}$ be a family of r-uniform hypergraphs. The *chromatic threshold* of $\mathcal{F}$ is the infimum of all non-negative reals c such that the subfamily of $\mathcal{F}$ comprising hypergraphs H with minimum degree at least $c\binom{|V(H)|}{r-1}$ has bounded chromatic number. This parameter has a long history for graphs ($r = 2$), and we begin its systematic study for hypergraphs.

Łuczak and Thomassé recently proved that the chromatic threshold of near bipartite graphs is zero, and our main contribution is to generalize this result to r-uniform hypergraphs. In an attempt to generalize Thomassen's result that the chromatic threshold of triangle-free graphs is 1/3, we prove bounds for the chromatic threshold of the family of 3-uniform hypergraphs not containing $\{abc, abd, cde\}$, the so-called generalized triangle.

In order to prove upper bounds we introduce the concept of *fiber bundles* and *fiber bundle dimension*, a structural property of fiber bundles that is based on the idea of Vapnik-Chervonenkis dimension in hypergraphs. Our lower bounds follow from explicit constructions, many of which use a generalized Kneser hypergraph. Using methods from extremal set theory, we prove that these generalized Kneser hypergraphs have unbounded chromatic number. This generalizes a result of Szemerédi for graphs and might be of independent interest. Many open problems remain.

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