

# The Typical Appearance of a Colouring of a Regular Bipartite Graph

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## Abstract

Consider a typical (uniformly chosen)  $q$ -colouring of an arbitrary regular bipartite graph. By symmetry, the expected proportion of vertices receiving a particular colour is  $1/q$ . The question to be addressed in this talk is: can anything more be said in general about the distribution of the proportion?

We show that if  $q$  is even, then the proportion is concentrated around  $1/q$ , while if  $q$  is odd then it is concentrated around the interval  $[1/(q+1), 1/(q-1)]$ , and we give examples to show that this interval is best possible.

When we are coloring the cube or the torus, we can say something much more precise: almost all  $q$ -colourings are obtained by partitioning the set of colours into two near-equal classes  $A$  and  $B$ , and colouring most of one bipartition class of the cube or torus from  $A$  and most of the other class from  $B$ . A nice corollary of this is a strong long-range influence result for uniformly chosen colourings of the cube or torus.

This is joint work with J. Engbers (Notre Dame). The approach is through entropy, building on J. Kahn's study of independent sets, and generalizes to arbitrary graph homomorphisms.