The maximum size of a Sidon set contained in a sparse random set of integers

Sangjune Lee
Emory University
slee242@emory.edu

A set $A$ of integers is a Sidon set if all the sums $a_1 + a_2$, with $a_1 \leq a_2$ and $a_1, a_2 \in A$, are distinct. In the 1940s, Chowla, Erdős and Turán showed that the maximum possible size of a Sidon set contained in $[n] = \{0, 1, \ldots, n-1\}$ is approximately $\sqrt{n}$. We study Sidon sets contained in sparse random sets of integers, replacing the ‘dense environment’ $[n]$ by a sparse, random subset $R$ of $[n]$.

Let $R = [n]_m$ be a uniformly chosen, random $m$-element subset of $[n]$. Let

$$F([n]_m) = \max\{|S|: S \subset [n]_m \text{ is Sidon}\}.$$ 

An abridged version of our results states as follows. Fix a constant $0 \leq a \leq 1$ and suppose $m = m(n) = (1 + o(1))n^a$. Then there is a constant $b = b(a)$ for which $F([n]_m) = n^{b+o(1)}$ almost surely. The function $b = b(a)$ is a continuous, piecewise linear function of $a$, not differentiable at two points: $a = 1/3$ and $a = 2/3$; between those two points, the function $b = b(a)$ is constant. This is joint work with Yoshiharu Kohayakawa and Vojtěch Rödl.