The maximum size of a Sidon set contained in a sparse random set of integers

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A set A of integers is a *Sidon set* if all the sums $a_1 + a_2$, with $a_1 \le a_2$ and $a_1, a_2 \in A$, are distinct. In the 1940s, Chowla, Erdős and Turán showed that the maximum possible size of a Sidon set contained in $[n] = \{0, 1, \ldots, n-1\}$ is approximately $\sqrt{n}$. We study Sidon sets contained in sparse random sets of integers, replacing the 'dense environment' $[n]$ by a sparse, random subset R of $[n]$.

Let $R = [n]_m$ be a uniformly chosen, random m-element subset of $[n]$. Let

$$F([n]_m) = \max\{|S| \colon S \subset [n]_m \text{ is Sidon}\}.$$

An abridged version of our results states as follows. Fix a constant $0 \le a \le 1$ and suppose $m = m(n) = (1 + o(1))n^a$. Then there is a constant $b = b(a)$ for which $F([n]_m) = n^{b+o(1)}$ almost surely. The function $b = b(a)$ is a continuous, piecewise linear function of a, not differentiable at two points: $a = 1/3$ and $a = 2/3$; between those two points, the function $b = b(a)$ is constant. This is joint work with Yoshiharu Kohayakawa and Vojtěch Rödl.