Rainbow Hamilton Cycles in Random Graphs

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Abstract

One of the most famous results in the theory of random graphs establishes that the threshold for Hamiltonicity in the Erdős-Rényi random graph $G_{n,p}$ is around $p \sim \frac{\log n + \log \log n}{n}$. Much research has been done to extend this to increasingly challenging random structures. In particular, a recent result by Frieze determined the asymptotic threshold for a loose Hamilton cycle in the random 3-uniform hypergraph by connecting 3-uniform hypergraphs to edge-colored graphs.

In this work, we consider that setting of edge-colored graphs, and prove a result which achieves the best possible first order constant. Specifically, when the edges of $G_{n,p}$ are randomly colored from a set of $(1 + o(1))n$ colors, with $p = \frac{(1+o(1)) \log n}{n}$, we show that one can almost always find a Hamilton cycle which has the further property that all edges are distinctly colored (rainbow).

Joint work with Alan Frieze.