Graphs with Average Degree Smaller Than $\frac{30}{11}$ Are Burning Slowly

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Abstract

We consider the following *firefighter problem* on a finite graph $G = (V, E)$. Suppose that a fire breaks out at a given vertex $v \in V$. In each subsequent time unit, a firefighter protects one vertex which is not yet on fire, and then the fire spreads to all unprotected neighbours of the vertices on fire. Since the graph is finite, at some point each vertex is either on fire or is protected by the firefighter, and the process is finished. The objective of the firefighter is to save as many vertices as possible.

Let (G, v) denote the number of vertices in G the firefighter can save when a fire breaks out at vertex $v \in V$, assuming the best strategy is used. The surviving rate $\rho(G)$ of G is defined as the expected percentage of vertices that can be saved when a fire breaks out at a random vertex of G, that is, $\rho(G) = \frac{1}{n^2} \sum_{v \in V} (G, v)$.

For any > 0, we show that a graph G on n vertices with at most $(\frac{15}{11} - \epsilon)n$ edges can be well protected, that is, $\rho(G) > \frac{\epsilon}{60} > 0$. Moreover, a construction of a random graph is proposed to show that the constant $\frac{15}{11}$ cannot be improved.