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Law of large numbers for an epidemic model

Let us describe two epidemic models (see [1]) which are called *frog models* (a model with one jump and a model with several jumps). In these models a discrete time random process is studied. In both models there is a number of particles. Some particles are active, other particles sleep. The dynamics of the process is described as follows. Active particles can jump on some other particles, and sleeping particles do not move. When a sleeping particle is hit by an active particle, it becomes active too.

Consider a set of points $\{x_1, ..., x_n\}$. At time 1 there is one particle in each point. There is an active particle in the point x_1. All the other particles are sleeping. Each active particle lives while it chooses points with inactive particles on them. It dies once it chooses to jump on a point which has been visited before by some active particle. The process continues until there are no more active particles.

Denote by $D_n(i)$ the number of alive sleeping particles at the moment i. Let σ_n be a moment of death of the last active particle. A limit distribution of the random variable $X_n = n - D_n(\sigma_n)$ is studied.

In the *model with one jump* at each time step only one active particle makes a jump (this active particle is uniformly chosen). A move of this particle is uniformly chosen too.

In 2010 F. Machado, H. Mashurian and H. Matzinger (see [2]) proved a central limit theorem for the described model.

Theorem 1. *Let q be the only solution of the equation $2p = -\ln(1-p)$, $p \in (0, 1)$, $\mu_r = 2 - \frac{1}{1-q}$, $\sigma = \sqrt{\frac{q-2q^2}{q-1}\frac{1}{\mu_r}}$. Then the convergence*

$$\frac{X_n - qn}{\sigma\sqrt{n}} \xrightarrow{d} N(0, 1)$$

holds.

In the *model with several jumps* at each time step every active particle is able to make a move. An active particle jumps with probability p independently of all other active particles. A move of this particle is uniformly chosen and doesn't depend on moves of other active particles. Note that if several particles jump on one point with a sleeping particle at the same time then all these particles survive.

A study of the distribution of X_n in the model with several jumps is much more difficult. We proved a law of large numbers for this random variable.

Theorem 2. *Let $f(n) = n^{3/4+\delta}$, $\delta > 0$. Then*

$$\frac{X_n - \mathsf{E}X_n}{f(n)} \xrightarrow{\mathsf{P}} 0, \quad n \to \infty.$$

References

[1] R. Durrett, *Random Graph Dynamics*, Cambridge University Press, New York, 2007.

[2] F. Machado, H. Mashurian, H. Matzinger, *CLT for the proportion of infected individuals for an epidemic model on a complete graph*, arXiV:1011.3601v1, 2010.